Devoir maison n°07 – mathématiques
Donné le 22/11/2017 – à rendre le 29/11/2017

Exercice 1

1. (a) $A = \dfrac{10a^2 x^3 y^2}{-4a^4 x^3 y} = -\dfrac{5y}{2a^2}$

   (b) $B = \dfrac{x^2}{x^2 - x} = \dfrac{x}{x - 1}$

   (c) $C = \dfrac{x^2 + x^3}{x^3 - x} = \dfrac{x + x^2}{x^2 - 1} = \dfrac{x(x+1)}{(x-1)(x+1)} = \dfrac{x}{x-1}$

   (d) $D = \dfrac{6x^2 - 4x}{9ax - 6a} = \dfrac{2x(3x-2)}{3a(3x-2)} = \dfrac{2x}{3a}$

2. (a) $E = \dfrac{2}{x+2} - \dfrac{1}{x-2} = \dfrac{2(x-2) - (x+2)}{(x+2)(x-2)} = \dfrac{x-6}{(x+2)(x-2)}$

   (b)

$$
\begin{aligned}
F &= \frac{1}{a(a+1)} - \frac{1}{a(a-1)} + \frac{2a}{a^2 - 1} \\
&= \frac{(a-1) - (a+1) + 2a \times a}{a(a+1)(a-1)} \\
&= \frac{2a^2 - 2}{a(a+1)(a-1)} \\
&= \frac{2(a-1)(a+1)}{a(a+1)(a-1)} \\
&= \frac{2}{a}
\end{aligned}
$$

   (c)

$$
\begin{aligned}
G &= \frac{x^3}{x^3 - x^2} + \frac{x}{x+1} - \frac{2x}{x^2 - 1} \\
&= \frac{x^3(x+1) + x \times x^2(x-1) - 2x \times x^2}{x^2(x-1)(x+1)} \\
&= \frac{x^4 + x^3 + x^4 - x^3 - 2x^3}{x^2(x-1)(x+1)} \\
&= \frac{2x^4 - 2x^3}{x^2(x-1)(x+1)} \\
&= \frac{2x^3(x-1)}{x^2(x-1)(x+1)} \\
&= \frac{2x}{x+1}
\end{aligned}
$$

3. (a) $H = \dfrac{\dfrac{x}{x-y} - \dfrac{y}{x+y}}{\dfrac{x}{x+y} + \dfrac{y}{x-y}} = \dfrac{\dfrac{x(x+y) - y(x-y)}{(x-y)(x+y)}}{\dfrac{x(x-y) + y(x+y)}{(x-y)(x+y)}} = \dfrac{x(x+y) - y(x-y)}{x(x-y) + y(x+y)} = \dfrac{x^2 + y^2}{x^2 + y^2} = 1$

(b)

$$I = \dfrac{\dfrac{x+1}{x-1} - \dfrac{x-1}{x+1}}{1 - \dfrac{x-1}{x+1}} = \dfrac{\dfrac{(x+1)^2 - (x-1)^2}{(x-1)(x+1)}}{\dfrac{(x+1)-(x-1)}{x+1}} = \dfrac{\dfrac{((x+1)-(x-1))((x+1)+(x-1))}{(x-1)(x+1)}}{\dfrac{2}{x+1}}$$

$$= \dfrac{\dfrac{2 \times 2x}{x-1}}{2} = \dfrac{2x}{x-1}$$

(c) $\quad J = \dfrac{\dfrac{a+b}{2} + \dfrac{b^2}{a-b}}{\dfrac{a+b}{2} - \dfrac{ab}{a+b}} = \dfrac{\dfrac{(a+b)(a-b) + 2b^2}{2(a-b)}}{\dfrac{(a+b)^2 - 2ab}{2(a+b)}} = \dfrac{a^2 + b^2}{2(a-b)} \times \dfrac{2(a+b)}{a^2 + b^2} = \dfrac{a+b}{a-b}$