The DemoNat project

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Introduction

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  - Analyse and validate proofs in natural language

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- Interest :
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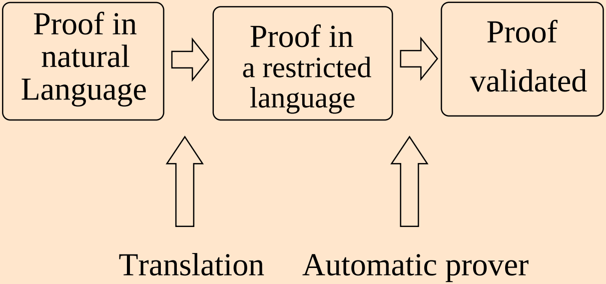
  - Simplicity
- Teams involved in the projet :
  - Lattice/Talana (Jussieu)
  - Calligramme (Nancy)
  - LAMA (Chambéry)

The system


Proof in natural Language
Proof in a restricted language
Proof validated
Translation
Automatic prover

My work in this project

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- Practical :
  - Definition of a restricted language
  - Implementation of a prover

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- Practical :
  - Definition of a restricted language
  - Implementation of a prover
- Theoretical :
  - ACGs and principal typing with two arrows
  - Study of a logic system observed from the prover

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  - Describes a proof
  - Uses a small grammar
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- Features :
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  - The grammar itself is independant from the logic
- Treatment :
  - Linked to a current goal
  - To each rule is associated a "trivial" goal
  - The nexts goals are given to the user

The grammar (a bit simplified) 1

nc

- ncs
- BY ... (WITH ...) ncs
- PROVE FORM nc MYIN nc
- BYABSURD HYPNAME nc
- SET EQUAL nc
- LABEL HYPNAME

ncs

- DEDUCE FORM nc
- TRIVIAL
- meta

The grammar (a bit simplified) 2

meta

LET CST meta
SEARCH VAR meta
ASSUME FORM meta
SHOW FORM nc SHOWN
meta MYTHEN meta
PBEGIN meta PEND
PROOF nc ENDPROOF

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PBEGIN (...) PEND : parenthesis

PROOF (...) ENDPROOF : proof of the current premiss

The justification

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- For each rule a formula is computed that justifies it
  - Share variables as much as possible
  - If no goal has changed, don't use the goal formula in the formula
- Formulas given with BY and WITH if not hypothesis
  - Are first proved
  - Are used as hints for the prover
  - Are forgotten in the next goals

The prover as a functor

The prover as a functor

```
module Prover : functor (Logic : Logic) -> 
sig
Exception Prove_fails

val prove : ( formula * int * constraints) list
                    -> formula
                    -> unit
(* raises Prove_fails when no proof is found *)
end
```

To have a prover :

- give a logic
- apply the functor to it.

Logic required

```
module type Logic =
sig
type formula (form)

val elim_all_neg : form -> form
...
type substitution (subs)

type constraints (csts)

val unif : csts -> form -> csts -> form ->
        int * subs * csts * form * form list

val get_rules : csts -> form -> bool ->
        (string * int * subs * csts * form list ) list
end
```

Resolution

- Principle : Finding a contradiction in a set of clauses (set of disjonctive formulas)
- Two rules
  - Resolution rule

$$\frac{C_1, L_1 \quad C_2, L_2 \quad \sigma = mgu(L_1, \overline{L_2})}{C_1\sigma, C_2\sigma}\, res$$

  - Contraction rule

$$\frac{C_1, L_1, L_2 \quad \sigma = mgu(L_1, L_2)}{C_1\sigma, L_1\sigma}\, contr$$

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- We don't want to decompose everything when we have $F \longrightarrow F$ to prove
- The idea :
  - use decomposition rules
  - clauses are sets of formulas
    (not necessarily atomic formulas)

Decomposition rules 2

Exple : Let $\{\neg F, \Gamma\}$ be a clause with $F = (A \to B)$
From $F \leftrightarrow (A \to B)$ we obtain two clauses :
$\{A, \Gamma\}$ and $\{\neg B, \Gamma\}$
It can be seen as resolutions with the following clauses on the literal $F \equiv X_1 \to X_2$:
$\{X_1, X_1 \to X_2\}$ and $\{\neg X_2, X_1 \to X_2\}$

$\longrightarrow$ Decomposing is making resolution with rule clauses.

$\longrightarrow$ **get_rules** asks for each formula which rules can be applied.

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  $\longrightarrow$ OL-deduction for clauses of splitting variables

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  - ...

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- Condition on the lexicon :

$$\mathcal{L}(c) \ : \ \mathcal{L}(\tau(c))$$

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- Thanks to the condition on the lexicon the mapping on types can be found thanks to a principal type algorithm

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The linear lambda calculus, useful while dealing with syntax, is limited in its expressiveness for semantics where one needs to write formulas using several occurences of a variable

So a calculus with two kind of arrows and variables was defined

While computing a principal type some problems appear

The calculus 1

$$\frac{}{\Gamma; \vdash c : \tau(c)}$$

$$\frac{}{\Gamma; x : \gamma \vdash x : \gamma} \qquad \frac{}{\Gamma, x : \gamma; \vdash x : \gamma}$$

$$\frac{\Gamma; \Delta, x : \alpha \vdash t : \beta}{\Gamma; \Delta \vdash \lambda^{\circ} x.t : \alpha \multimap \beta} \qquad \frac{\Gamma, x : \alpha; \Delta \vdash t : \beta}{\Gamma; \Delta \vdash \lambda x.t : \alpha \rightarrow \beta}$$

The calculus 2

$$\frac{\Gamma; \Delta_1 \vdash t : \alpha \multimap \beta \quad \Gamma; \Delta_2 \vdash u : \alpha}{\Gamma; \Delta_1, \Delta_2 \vdash (t\ u) : \beta}\ (*)$$

$$\frac{\Gamma; \Delta \vdash t : \alpha \rightarrow \beta \quad \Gamma; \vdash u : \alpha}{\Gamma; \Delta \vdash (t\ u) : \beta}$$

$(*)$ $Dom(\Delta_1) \cap Dom(\Delta_2) = \emptyset$

The principal typing

- We need a typing rule scheme

$$\frac{\Gamma; \Delta \vdash t : \alpha \multimap ?_n \beta \quad \Gamma; \vdash u : \alpha}{\Gamma; \Delta \vdash (t\ u) : \beta}$$

The principal typing

- We need a typing rule scheme

$$\frac{\Gamma; \Delta \vdash t : \alpha {-?_n} \beta \quad \Gamma; \vdash u : \alpha}{\Gamma; \Delta \vdash (t\ u) : \beta}$$

- usual typing algorithm (Damas-Milner) with constraints while typing application $(u\ v)$:
  - if v has free linear variables u must have type $\multimap$
  - overwise we take a new unspecified arrow $-?$ to type u

- Generally some unspecified arrows are remaining
- If we want to avoid them, there can be some problems
- Example :

  let

  $$t = \lambda g \lambda f \mathring{\lambda} x \lambda u.(g \quad (f\ x) \quad (f\ \lambda t.(t\ u)))$$

  its principal type is

  $$\begin{aligned} \vdash t\ :\ (b \multimap b {-?_1}\, n) &\quad \rightarrow \\ (((a {-?_2}\, e) \rightarrow e) \multimap b) &\quad \rightarrow \\ ((a {-?_2}\, e) \rightarrow e) &\quad \multimap \\ a &\quad \rightarrow \quad n \end{aligned}$$

  t is neither linear nor η-long

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- η-long terms have the arrow property

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- Linear terms :
  - Generalize the property :
    1. each type variable appearing appears twice
       with a positive occurrence
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  - true for terms in β-normal form
  - it is stable under β-expansion

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  - The typing algorithm is adapted to η-long terms

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    1. everything is justified (by justifying terms)
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Proofs (very general ideas) 2

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  - If t is an η-long term with a negative $\rightarrow$ then this arrow can be replaced by a $\multimap$

Conclusion / Projects

- Practical for the prover :
  - Needs constant improvements
    functions for weight, data structures, strategies,...
  - Has been used by two classical logics
    propositional and first order
  - Will be used in PhoX, proof assistant
    developped by C. Raffalli
- Theoretical :
  - In the ACGs :
    - Work on the matching problem
      I. Cervesato defined similar calculus
    - Find another proof for the principal typing with subtypes
    - Work on another calculus, with features
  - For the prover :
    - Define a logic system to prove theoretical things
      a system between free deduction of M. Parigot
      and the calculus of structures of A. Guglielmi