Proofs using PhoX

and new_command

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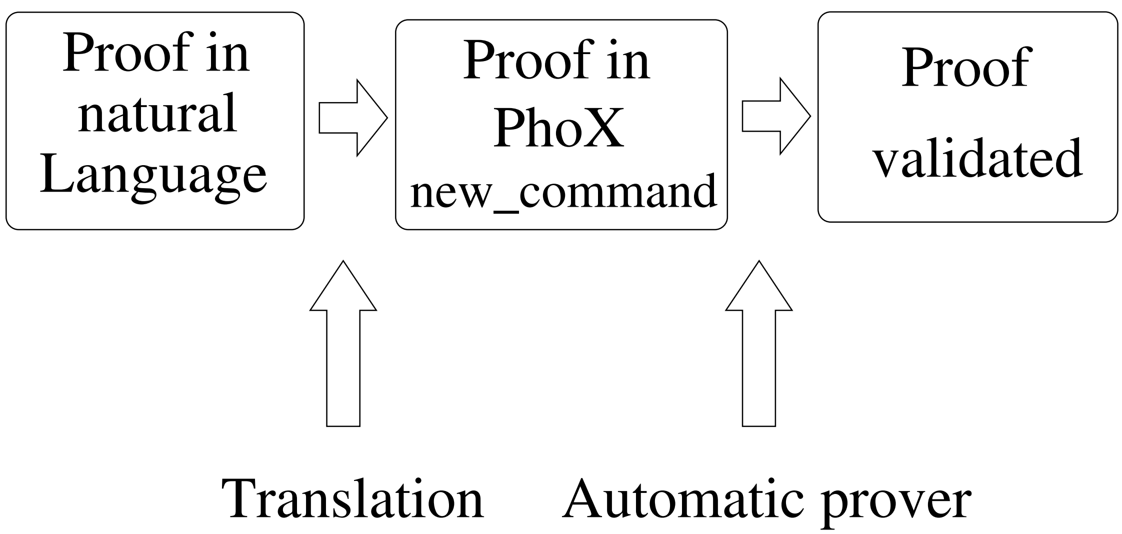
Introduction

- Aim of the **DemoNat** project :
  Analyse, validade proofs in natural language
- Interests of this project :
  - For students
  - Less theory needed
- Who works on this project :
  Laboratories of linguistics and mathematics
  - Lattice / TaLaNa (Jussieu)
  - Calligramme (Nancy)
  - LaMa (Chambéry)


Proof in natural Language
Proof in PhoX new_command
Proof validated
Translation
Automatic prover

New_command : Syntax

let .. cmd (naming)

| **assume** .. { **and** ..} cmd

| **deduce** .. { **and** ..} cmd

| **by** .. { **with** ..} cmd (hints)

| **show** .. (replace goal)

| **prove** .. (cut rule)

| **trivial**

| $\emptyset$

| cmd **then** cmd (cases)

| **begin** cmd **end** (brackets)

New_command : a Meta-rule

$$\mathcal{H}\ \frac{H_1,\ldots,H_n \vdash\ B_1 \qquad \ldots \qquad K_1,\ldots,K_m \vdash\ B_s}{\vdash\ B}$$

The translation

M, N topological spaces. $f : M \longrightarrow N$

2 notions of continuity :
- global : continous.
- local : continuous_at.

Property :
f continuous $\rightarrow \forall m \in M\ f$ continuous at m.
proof :

Adding assumptions & changing the goal

$$\boxed{\begin{array}{l} \vdash f \text{ continuous} \\ \quad \to \forall m \in M \quad f \text{ continuous\_at } m \\ \\ \text{Assume } f \text{ is continuous, let } m \in M. \end{array}}$$

$$\Downarrow$$

assume f continuous let $m \in M$
show f continuous_at m.

finding formal definitions

$$\begin{array}{rl} \text{H} := & f \text{ continuous} \\ \text{H0} := & M\ m \\ \vdash & f \text{ continuous\_at } m \end{array}$$

Let V a neighbourhood of $f(m)$, we must prove that **it**'s reverse image is a neighbourhood of m.

$$\Downarrow$$

let V assume V neighbourhood.N $(f\ m)$
show (**reverse** f $\mathbf{V}$) neighbourhood.M m.

finding assumptions

$$\boxed{\begin{array}{rl} \text{H} := & f \text{ continuous} \\ \text{H0} := & M\ m \\ \text{H1} := & V \text{ neighbourhood.N } (f\ m) \\ \multicolumn{2}{l}{\vdash (\text{reverse } f\ V) \text{ neighbourhood.M } m} \\ \\ \multicolumn{2}{l}{\textbf{By definition of a neighbourhood}\text{,}} \\ \multicolumn{2}{l}{\text{let } O \text{ an open set}} \\ \multicolumn{2}{l}{\text{included in } V \text{ and containing } f(m).} \end{array}}$$

$$\Downarrow$$

by H1 let O
assume O open.N and $O \subset V$ and O $(f\ m)$.

H :=	f continuous
$\vdots$	
H2 :=	O open.N
H3 :=	$O \subset V$
H4 :=	$O\ (f\ m)$

$\vdash$ (reverse $f\ V$) neighbourhood.M m

As f is continuous, $f^{-1}(O)$ is open and $m \in f^{-1}(O)$.

$$\Downarrow$$

by f continuous deduce
(reverse $f\ O$) open.M and (reverse $f\ O$) m.

Assumption or not assumption

$$\begin{array}{rl} & \vdots \\ \text{H3} := & O \subset V \\ & \vdots \\ \text{H5} := & (\text{reverse } f\ O) \text{ open.M} \\ & \wedge\ (\text{reverse } f\ O)\ m \end{array}$$

$\vdash$ (reverse f V) neighbourhood.M m

As $f^{-1}(O) \subset f^{-1}(V)$, the proof is finished.

$$\Downarrow$$

deduce (reverse f O) $\subset$ (reverse f V) trivial.

Proofs of formulas

$$\forall i \in I \quad A \cap h(i) = f(i) \Rightarrow A \cap \bigcap_{i \in I} h(i) = \bigcap_{i \in I} f(i)$$

$$\boxed{\begin{array}{l} \quad \mathsf{E0} := \quad \forall i \in I \quad A\cap\ h\ i = f\ i \\ \qquad\qquad \vdash \quad A\cap\ (\mathsf{Inter}\ h\ I) \subset (\mathsf{Inter}\ f\ I) \\ \\ \mathsf{let}\ m \in A\cap\ (\mathsf{Inter}\ h\ I)\ \mathsf{show}\ (\mathsf{Inter}\ f\ I)\ m. \end{array}}$$

$$\downarrow$$

$$\boxed{\begin{array}{lll} \mathsf{E0} := & \forall i \in I \quad A\cap\ h\ i = f\ i & \\ \qquad \vdash & \forall m\ [(A\cap\ (\mathsf{Inter}\ h\ I))\ m & \\ & \qquad \to (\mathsf{Inter}\ f\ I)\ m] & \\ & \to & \\ & A\cap\ (\mathsf{Inter}\ h\ I) \subset (\mathsf{Inter}\ f\ I) & \end{array}}$$

$$\downarrow$$

$$\boxed{\begin{array}{lll} \mathsf{E0} := & \mathsf{E0} & \\ \qquad \vdash & \mathsf{K} \to \mathsf{K} & \end{array} \quad \text{Up to a definition}}$$

F closed, $(x_n)_n \subset F$
$(x_n)_n$ converging to $x0 \to x0 \in F$

$$
\boxed{\begin{array}{ll}
 & \vdots \\
\mathsf{G} := & \forall V \text{ neighbourhood } x0 \\
 & \exists n \in \mathbf{N} \quad V\ (x\ n) \\
\mathsf{H} := & V \text{ neighbourhood } x0 \\
\vdash & {\color{red}\exists y\ (F \cap V) y} \\
 & \\
\multicolumn{2}{l}{\text{by [G] with [H] let } n \in \mathbf{N} \text{ assume } {\color{blue}V\ (x\ n)}.}
\end{array}}
$$

$$\downarrow$$

$$
\boxed{\begin{array}{ll}
\vdots & \\
\vdash & \forall n \in \mathbf{N}\ ({\color{blue}V\ (x\ n)} \to {\color{red}\exists y\ (F \cap V)\ y}) \\
 & \to {\color{red}\exists y\ (F \cap V)\ y}
\end{array}}
$$

$$\downarrow$$

$$
\boxed{\begin{array}{ll}
\mathsf{G} := & \forall\ V\ (H[V] \to \\
 & \qquad \exists n\ (n \in \mathbf{N} \wedge V\ (x\ n))) \\
\mathsf{H} := & H[V] \\
\vdash & \forall n\ (n \in \mathbf{N} \to {\color{blue}V\ (x\ n)} \to {\color{red}K}) \to {\color{red}K}
\end{array}}
$$

$$C_1 := \{\forall V\ (H[V] \rightarrow \exists n\ (n \in \mathbf{N} \wedge V\ (x\ n)))\}$$
$$C_2 := \{H[V]\}$$
$$B := \{\neg\ [\forall n\ (n \in \mathbf{N} \rightarrow V\ (x\ n) \rightarrow K) \rightarrow K]\}$$

$$\downarrow$$

$$C_1 := \{\forall V\ (H[V] \rightarrow \exists n\ (n \in \mathbf{N} \wedge V\ (x\ n)))\}$$
$$C_2 := \{H[V]\}$$
$$C_3 := \{\forall n\ (n \in \mathbf{N} \rightarrow V\ (x\ n) \rightarrow K)\}$$
$$G := \{\neg K\}$$

$$\downarrow$$

$$C_1' := \{(H[V?] \rightarrow \exists n\ (n \in \mathbf{N} \wedge V?\ (x\ n)))\}$$
$$C_2 := \{H[V]\}$$
$$C_3' := \{\neg n? \in \mathbf{N} \vee \neg V\ (x\ n?) \vee K)\}\}$$
$$G := \{\neg K\}$$

$$\downarrow$$

$$\boxed{\begin{array}{l} C_4 := \{\exists n\ (n \in \mathbf{N} \wedge V\ (x\ n))\} \\ C_3' := \{\neg n? \in \mathbf{N} \vee \neg V\ (x\ n?) \vee K)\}\} \\ G := \{\neg K\} \end{array}}$$

$$\downarrow$$

$$\boxed{\begin{array}{l} C_4' := \{n \in \mathbf{N} \wedge V\ (x\ n)\} \\ C_3' := \{\neg n? \in \mathbf{N} \vee \neg V\ (x\ n?) \vee K)\}\} \\ G := \{\neg K\} \end{array}}$$

$$\downarrow$$

$$\boxed{\begin{array}{l} C_5 := \{n \in \mathbf{N}\} \\ C_6 := \{V\ (x\ n)\} \\ C_3' := \{\neg n? \in \mathbf{N} \vee \neg V\ (x\ n?) \vee K)\}\} \\ G := \{\neg K\} \end{array}}$$

$$\downarrow$$

$$\square$$

Conclusion

- bests results by reading the whole proof
- Hints given to help the proof
- Inverse resolution method in a lazy way

The link to the project (in french) :
http ://demonat.linguist.jussieu.fr/

See my page at
http ://www.lama.univ-savoie.fr/$\sim$ thevenon/